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A Game Theory Perspective on Fiduciary Finance

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Abstract

Fiduciary finance is finance facilitated through the fiduciary relationship, typically intermediated by the institutional trustee or trust institution. In general, the trustee receives property from the settlor and holds it to or for the benefit of the beneficiary in the trust, where a savings-investment transfer is made between surplus and deficit units. Presenting a generic model, this paper clarifies the economic characteristics of fiduciary finance, by means of focusing on the underlying principal-agent relationship between the settlor and the trustee in the self-benefit trust, in the light of the equilibrium properties of Cournot and Stackelberg duopoly games.

Keywords: Trust; Active trust; Passive trust; Fiduciary relationship; Principal-agent relationship; Direct and intermediated finance; Financial intermediaries

Introduction

This paper investigates the economic characteristics of fiduciary finance defined as finance through the fiduciary relationship, presenting a generic model based on the game theory of duopoly after the preceding attempts made by Nishiyama (2006; 2008; 2012). Specifically, the paper focuses on the process of financial intermediation by the institutional trustee or trust institution as today's typical fiduciary in the light of the equilibrium properties of Cournot (1838) and Stackelberg (1934) duopoly games.

A trust is defined as "a fiduciary relationship with respect to property" (Restatement Third, Trusts §2). It is created by the settlor legally transferring property to the trustee who holds it in trust only to or for the benefit of the beneficiary. The beneficiary and trustee are referred to as the principal and her agent in the principal-agent relationship, which is abstracted as the economic essence of the fiduciary relationship.

Moreover, in accordance with the intent of the settlor, a trust is active, if the trustee has affirmative duties to perform, and it is passive, if her responsibilities are entirely negative (Restatement Third, Trusts §6, §6 Comment c). As Nishiyama (2018; 2019) clarified, the choice between active and passive trusts has the effect of the distribution of discretionary power among trust parties as a key determinant for the decision-making in the trust. While the active trust allows the trustee full discretion over trust management and investment, the passive trust provides her with no discretion. In the passive trust, the property held therein by the trustee with negative responsibilities is managed based on directions from the settlor or the person entrusted with the power to give directions by the settlor, where the settlor or the entrusted person has power to control the action of the trustee (Trust Business Act §§2 (3), 65, Act No. 154 of 2004; Japan).

Although a number of studies have been carried out on the fiduciary agents including the institutional trustee, in particular their activities as institutional investors, little analytical research on them has been hitherto conducted with relevance to the inherent fiduciary relationship thereof. This paper provides an approach to fiduciary finance by means of setting forth the concept of self-benefit private trust, where the settlor acts simultaneously as the beneficiary, thus having the exclusive right to receive all income from the trust. Finance, in general, facilitates the savings-and-investment process between surplus units (suppliers of funds), deficit units (borrowers of funds) and financial intermediaries as intermediating units. In the paper, fiduciary finance is simply arranged under the self-benefit trust settled by a surplus unit for her own benefit with the institutional trustee as a financial intermediary. Besides, the settlor's choice between active and passive trusts determines the financial method in fiduciary finance. According to whether the trust is active or passive, the institutional trustee manages and invests the property transferred from the surplus unit with or without discretionary power. Since the institutional trustee is vested with the discretionary power in the active trust, she as a financial intermediary thus engages in intermediated finance as rigorously defined. On the other hand, she exercises no discretionary power in the part of her financial intermediation in the passive trust, where her management and investment of trust property are made at the directions of her settlor, substantially providing a route for direct finance in the intermediated form.

The Model

Let there be a settlor, who is also a beneficiary, transferring her property to an institutional trustee to create a self-benefit trust for her own benefit. There are the two types of trusts, active and passive trusts, indexed by $i \in \{A, P\}$, where a part of the trust property is held actively at the trustee's discretion and the other part is held upon passive trust with her negative responsibilities, represented by $x_i \in R_+$, $i \in \{A, P\}$, respectively. Note that the characteristics of the trust property give it a degree of asset specificity which represents a segment of the financial system.

The whole property is

$$X = \sum_{i \in \{A,P\}} x_i \text{ .} \tag{1}$$

The trust property is managed and invested in costless finance with the inverse demand function

$$r = r(X), \tag{2}$$

where r is the interest rate or net return rate. In the model,

$$\exists X_0 \in R_{++} \left[\forall X < X_0 : r'(X) < 0, \forall X \geq X_0 : r(X) = 0 \right] \tag{A3}$$

is assumed on (2) for inner solution, with the "A" put before the equation number referring to "assumption."

Since the cost function is negligible, the profit or net return from the investment is formulated as

$$\pi_i \equiv x_i r(X) = x_i r\left(x_i + x_j\right),\ i, j \in \{A, P\},\ i \neq j \text{ .} \tag{4}$$

The office of the trustee with an ability to grasp all the flows of trust investment is assumed to generally provide the active trust with an advantage in making her optimal investment decision. Hence, the trustee chooses to act such that the active and passive trusts sequentially decide how much property each trust holds to manage and invest (Stackelberg game) or that both trusts decide independently of each other and at the same time (Cournot game). The investment of the trust property held passively, or the size of the passive trust, is thus determined such that the following first order condition

$$\frac{\partial \pi_P}{\partial x_P} = r\left(x_A + x_P\right) + x_P \frac{\partial r}{\partial x_P} = 0 \tag{5}$$

is satisfied. The second order condition

$$\frac{\partial^2 \pi_P}{\partial x_P^2} = 2\frac{\partial r}{\partial x_P} + x_P \frac{\partial^2 r}{\partial x_P^2} < 0 \tag{A6}$$

is assumed to be satisfied. Solving (5) with respect to x_P, the property held in passive trust is the function of that in active trust as

$$x_P \equiv \phi\left(x_A\right) = \underset{x_P \in (0, X_0)}{\arg\max}\left[x_P r\left(x_A + x_P\right)\right], \tag{7}$$

where

$$\phi'\left(x_A\right) = \frac{dx_P}{dx_A} = -\left(\frac{\partial r}{\partial x_A} + x_P \frac{\partial^2 r}{\partial x_A \partial x_P}\right) \Big/ \left(2\frac{\partial r}{\partial x_P} + x_P \frac{\partial^2 r}{\partial x_P^2}\right) < 0 \text{ (resp. } > 0\text{)}$$

$$\text{as } \frac{\partial r}{\partial x_A} + x_P \frac{\partial^2 r}{\partial x_A \partial x_P} < 0 \text{ (resp. } > 0\text{)}. \tag{8}$$

(7) is applied as the function of trust–acceptance. In the light of (A3) and (5),

$$\frac{\partial r}{\partial x_A} = \frac{\partial r}{\partial x_P} = r'\left(X\right) < 0 \tag{9}$$

and obviously,

$$\frac{\partial^2 r}{\partial x_A^2} = \frac{\partial^2 r}{\partial x_P^2} = \frac{\partial^2 r}{\partial x_A \partial x_P} = \frac{\partial^2 r}{\partial x_P \partial x_A} = r''\left(X\right), \tag{10}$$

whence

$$1 + \phi'\left(x_A\right) > 0 \tag{11}$$

regardless of the sign of (8), i.e., whether the function (7) is decreasing or increasing.

When the trustee chooses to play the Cournot game with respect to trust property, the investment of the property held upon active trust, x_A, is determined in the same way as that of passive trust, x_P, reaching the Cournot-Nash equilibrium through the interaction between the two types of trusts. At the equilibrium in the Cournot game,

$$x_i^C = \phi\left(x_j^C\right),\ i, j \in \left\{A, P\right\},\ i \neq j, \tag{12}$$

where the superscripted "*C*" denotes the Cournot-Nash equilibrium. In the model of this paper, as the two types of trusts are facing the same demand function and both have no cost function, their respective equilibrium investments satisfy

$$x_A^C = x_P^C = x^C. \tag{13}$$

The whole property to be managed and invested is

$$X^C = \sum_{i \in \{A,P\}} x_i^C = x_A^C + x_P^C = 2x^C \tag{14}$$

and, inserting (14) into (2), the interest rate is

$$r^C = r\left(X^C\right), \tag{15}$$

which also leads to

$$\pi_i^C = x_i^C r\left(x_i^C + x_j^C\right) = x_i^C r\left(X^C\right) = x^C r^C = \pi^C \text{, } i, j \in \{A, P\} \text{, } i \neq j \text{.} \tag{16}$$

On the other hand, the active and passive trusts sequentially determine their respective investment to maximize their profit (4) in the Stackelberg game. Hence, the active trust decides how much to hold to invest, taking account of the size of the property upon passive trust (7). Mathematically, the ordinary differentiation with respect to x_A is applied to the maximization of the profit thereon. Thus, the first order condition to be satisfied is

$$\frac{d\pi_A}{dx_A} = r\left(x_A + \phi\left(x_A\right)\right) + x_A\left(\frac{\partial r}{\partial x_A} + \frac{\partial r}{\partial x_P}\phi'\left(x_A\right)\right) = 0 \text{.} \tag{17}$$

To ensure a unique solution, assume that the second order condition

$$\begin{aligned}\frac{d^2\pi_A}{dx_A{}^2} &= 2\left(\frac{\partial r}{\partial x_A} + \frac{\partial r}{\partial x_P}\phi'\left(x_A\right)\right) \\ &\quad + x_A\left[\frac{\partial^2 r}{\partial x_A^2} + 2\frac{\partial^2 r}{\partial x_A \partial x_P}\phi'\left(x_A\right) + \frac{\partial^2 r}{\partial x_P^2}\left(\phi'\left(x_A\right)\right)^2 + \frac{\partial r}{\partial x_P}\phi''\left(x_A\right)\right] \\ &< 0\end{aligned} \tag{A18}$$

holds.

At the Stackelberg equilibrium, the investment of the property upon active trust, denoted by x_A^L, with Stackelberg leadership as the first-mover, is formulated as

$$x_A^L \equiv \underset{x_A \in (0, X_0)}{\arg\max}\left[x_A r\left(x_A + \phi\left(x_A\right)\right)\right]. \tag{19}$$

Meanwhile, the investment of the property held passively, x_P^F, as follower or the second-mover, is determined by (7) as

$$x_P^F = \phi\left(x_A^L\right). \tag{20}$$

The superscripts "*L*" and "*F*" refer to Stackelberg leader and follower, respectively.

Hence, similarly to (14) and (15), the whole property and the interest rate are

$$X^S = x_A^L + x_P^F = x_A^L + \phi\left(x_A^L\right) \tag{21}$$

and

$$r^S = r\left(X^S\right). \tag{22}$$

where the superscripted "*S*" denotes the Stackelberg equilibrium. The optimized profits of the active and passive trusts, as leader and follower, are respectively

$$\pi_A^L = x_A^L r\left(x_A^L + x_P^F\right) = x_A^L r\left(X^S\right) = x_A^L r^S \tag{23}$$

and

$$\pi_P^F = x_P^F r\left(x_P^F + x_A^L\right) = x_P^F r\left(X^S\right) = x_P^F r^S . \tag{24}$$

The Two Types of Trusts in the Light of the Equilibrium Properties

This section examines and clarifies the two types of trusts in the light of the equilibrium properties of Cournot and Stackelberg games. We focus on the property held to be invested and the profit thereon in the two types of trusts, because of limited space.

To begin with, we consider and compare the trust property held in the two types of trusts to be managed and invested. At the Stackelberg equilibrium, in view of (5), (7), (17), (19) and (20),

$$\left.\frac{d\pi_A}{dx_A}\right|_{x_A^L} = r\left(X^S\right) + x_A^L\left(1+\phi'\left(x_A^L\right)\right)r'\left(X^S\right) = 0 \tag{25}$$

and

$$\left.\frac{\partial\pi_P}{\partial x_P}\right|_{x_P^F} = r\left(X^S\right) + x_P^F r'\left(X^S\right) = 0 . \tag{26}$$

Associating (25) and (26),

$$x_A^L - x_P^F = -x_A^L\phi'\left(x_A^L\right), \tag{27}$$

whence

$$x_A^L > x_p^F \text{ (resp. } x_A^L < x_P^F\text{), if } \phi'\left(x_A\right) < 0 \text{ (resp. } > 0\text{).} \tag{28}$$

With the Cournot-Nash equilibrium put together in view of (5), the ordinary differentiation of π_A at x^C results in

$$\left.\frac{d\pi_A}{dx_A}\right|_{x^C} = x^C r'\left(X^C\right)\phi'\left(x^C\right) > 0 \text{ (resp. } < 0\text{), if } \phi'\left(x^C\right) < 0 \text{ (resp. } > 0\text{).} \tag{29}$$

Since π_A is strictly concave with respect to x_A on (A18), the differentiation (29), coupled with (25), leads to

$$x_A^L > x^C \text{ (resp. } x_A^L < x^C\text{) as } \left.\frac{d\pi_A}{dx_A}\right|_{x^C} > 0 \text{ (resp. } < 0\text{), if } \phi'\left(x^C\right) < 0 \text{ (resp. } > 0\text{).} \tag{30}$$

Meanwhile, (12), (13), (20) and (30) produce

$$x_P^F - x^C = \phi\left(x_A^L\right) - \phi\left(x^C\right) = \left(x_A^L - x^C\right)\phi'\left(\overline{x}\right) < 0 , \tag{31}$$

where $\overline{x} \in \left(x_A^L, x^C\right)$ serves as an intermediate value. Hence, regardless of the sign of $\phi'\left(\overline{x}\right)$,

$$x_P^F < x^C . \tag{32}$$

Since $x_A^C = x_P^C = x^C$ from (13), the results (28), (30) and (32) are combined to yield

$$x_A^L > x^C > x_p^F \text{ (resp. } x_A^L < x_P^F < x^C\text{), if } \phi'(x_A) < 0 \text{ (resp. } > 0\text{)}. \tag{33}$$

For the whole property, as (14), (21) and (31) result in

$$X^S - X^C = \left(x_A^L + \phi\left(x_A^L\right)\right) - \left(x^C + \phi\left(x^C\right)\right) = \left(x_A^L - x^C\right)\left(1 + \phi'\left(\overline{x}\right)\right). \tag{34}$$

where $1 + \phi'\left(\overline{x}\right) > 0$ from (11), we have in the light of (33),

$$X^S > X^C \text{ (resp. } X^S < X^C\text{) as } x_A^L > x^C \text{ (resp. } x_A^L < x^C\text{),}$$

$$\text{if } \phi'(x_A) < 0 \text{ (resp. } > 0\text{)}. \tag{35}$$

On the other hand,

$$r^S - r^C = r\left(X^S\right) - r\left(X^C\right) = \left(X^S - X^C\right) r'\left(X^*\right), \tag{36}$$

where $X^* \in \left(X^C, X^S\right)$ is an intermediate value. Hence, taking account of (A3) and (35),

$$r^S < r^C \text{ (resp. } r^S > r^C\text{) as } X^S > X^C \text{ (resp. } X^S < X^C\text{),}$$

$$\text{if } \phi'(x_A) < 0 \text{ (resp. } > 0\text{)}. \tag{37}$$

Let us proceed to the analysis and comparison of the profit on trust investment. In view of (23) and (24),

$$\pi_A^L - \pi_P^F = \left(x_A^L - x_P^F\right) r\left(X^S\right). \tag{38}$$

(33) and (38) lead to

$$\pi_A^L > \pi_P^F \text{ (resp. } \pi_A^L < \pi_P^F\text{) as } x_A^L > x_P^F \text{ (resp. } x_A^L < x_P^F\text{),}$$

$$\text{if } \phi'(x_A) < 0 \text{ (resp. } > 0\text{)}. \tag{39}$$

In the same way as (30), equations (25) and (29) result in

$$\pi_A^L > \pi^C, \tag{40}$$

regardless of the sign of (8) or the slope of (7).

In the case that the function (7) is downward, combined with $x_A^L > x^C > x_p^F$ in (33) and $r^S < r^C$ in (37),

$$\pi^C = x^C r^C > x^C r^S > x_P^F r^S = \pi_P^F, \tag{41}$$

thereby, taking account of (39) and (40),

$$\pi_A^L > \pi^C > \pi_P^F. \tag{42}$$

If the reaction function is upward, it is straightforward from (39) and (40) that

$$\pi^{C} < \pi_{A}^{L} < \pi_{P}^{F}\,. \tag{43}$$

Conclusion

In this paper, we have provided a game-theoretic analysis of fiduciary finance and its economic characteristics in the light of Cournot and Stackelberg duopoly games. Thereby, the management and investment of trust property and their optimization have been investigated as relevant to the equilibrium properties of these games.

Above all, the preceding section has clarified the relation with respect to trust property and profit between the two types of trusts or the methods of financial intermediation by the institutional trustee formulated as Cournot and Stackelberg games. This relation is decided first by whether the trust-acceptance function (7) is decreasing or increasing, then by whether the institutional trustee acts as the first-mover (Stackelberg leader) or not. Among the central results of the paper are (33), (35), (36), (42) and (43). These results also suggest that the trustee may not always have incentives for risky behavior and moral hazard.

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